

# MATH OLYMPICS

**Format:** In-Person

**Registration Guidelines:**

1. A limit of two (2) students from each team may register for this competition.
2. Each student competes individually. As such collaboration is not permitted.

**Procedure:**

1. No knowledge of Trigonometry or Calculus is required. An understanding of only Geometry, Algebra I & II and Statistics & Probability are sufficient for a student.
2. A number 2 pencil is required.
3. Basic scientific calculators are allowed. Calculators with graphing ability, such as the TI-83 or higher, are NOT allowed.
4. Students will get a set of problems in different rounds. The student with the highest number of points moves on to the next round.
5. A ballot is not provided for this competition, as scoring will be based solely on test performance.
6. Electronic device usage is not permitted during the competition.

**Layout:**

1. The competition will be split up into three portions.
  - a. The first portion will be the MIST Math Olympics Pretest.
  - b. The second portion will be the CIPHERING Round I.
  - c. The last portion will be the CIPHERING Round II.
  - d. If two or more competitors tie by the end of CIPHERING Round II, then the competitors will go into the CIPHERING Bonus Round.
2. The Pretest will consist of 30 different questions ranging from Geometry and Algebra and will be one hour long.
3. The two CIPHERING Rounds will both contain 10 questions each of which will be given two minutes.
  - a. Anyone that answers the question correctly in the given time will be awarded a point.
  - b. Additionally the first finisher with the correct answer of each question will receive 4 additional points for a total of 5, second will receive 3, third 2, and fourth will receive 1.
4. The cIPHERING round questions are not only designed to be difficult but also put the competitor under time pressure by giving him/her only a couple minutes to answer each question.

**Notes:**

1. The problems will be rated from (0,1,2,3) with 0 being easy, 1 being fair, 2 being difficult, and 3 being challenging.
  - a. Type zero (0) questions will only be found on the pretest.
  - b. Type one (1) may be found on the pretest or CIPHERING I.
  - c. Type two (2) may be found on the last couple questions of the pretest, or CIPHERING Rounds I and II.
  - d. Type three (3) questions may be found in CIPHERING Round II or the CIPHERING Bonus Round.

# MATH OLYMPICS STUDY GUIDE

- 1. Geometric Formulas (The competitor will be asked to recognize or apply these formulas)**
  - a. Area and Perimeter of two dimensional shapes
  - b. Volume and Surface Area of three dimensional shapes
  - c. Arc length formula
  - d. Slope of Line
  - e. Midpoint and Distance Formula
  - f. Equation of a circle
- 2. Geometry (The competitor needs to have a strong understanding of these topics)**
  - a. Find the point of intersection between two lines
  - b. Find a line parallel/perpendicular to a given line
- 3. Graphing (The competitor will be asked to apply these methods)**
  - a. Graphing a function
  - b. From a given graph determine if the equation is a function
  - c. Identify an odd or even function
- 4. Algebraic Formulas and Methods (The competitor will be asked to apply these methods)**
  - a. Quadratic Formula
  - b. Pascal's Triangle
  - c. Finding a quadratic equation from given points
  - d. Factoring
  - e. Simplifying polynomial expressions
  - f. Simplifying expression with radicals
  - g. Finding max and min
  - h. Describing the nature of the roots
  - i. Given  $g(x)$  and  $f(x)$ , then find  $f(g(x))$ .
- 5. Algebra (The competitor needs to have a strong understanding of these topics)**
  - a. Mean, Median, and Mode
  - b. Logarithmic functions
  - c. Inverse functions
  - d. Range and Domain
  - e. Basic rate functions
  - f. Factorials
  - g. Inequalities
- 6. Probability and Statistics**
  - a. Combination
  - b. Binomial Probability Formula
  - c. Mean, median, mode and standard deviation
- 7. Trigonometry**
  - a. Triangles
  - b. Sines, cosines, and tangents
  - c. Pythagorean Theorem and Identities
- 8. Logic**
  - a. Puzzles
  - b. Set Theory
  - c. Model Theory

# MATH OLYMPICS QUESTION GUIDE

## Example Geometric Formula Questions:

1. The circumference of a circle is  $8\pi$ , find the area of the circle. (0)
2. Calculate the volume + surface area of a box with the dimensions:  $l = 30\text{cm}$ ,  $w = 5\text{cm}$ ,  $h = 10\text{cm}$ . (1)
3. A right triangle has a hypotenuse which measures 10cm. If one of the sides measures 8cm, then find the length of the other side. (0)
4. There exists an arc with the length 6mcm which is formed by the angle of  $270^\circ$ . Find the radius. (1)
5. A line passes through the point (4,5) and (-2,8). Find the equation of the line. (0)
6. There exists a line segment with its endpoints at (-1,-2) and (4,8). Find the midpoint of the line segment. (0)
7. A circle has the radius of 8 with its center at (2,4). Find the equation of the circle. (0)
8. Find the distance between the points (1,2) and (6,7). Leave the answer in simplest radical form. (1)
9. Given the points A (1,-3) and B (-5,11), find the equation of the line perpendicular to the line segment AB which crosses at the midpoint of AB. (2)

## Example Geometry Questions:

1. Find the distance between the lines  $y = 3x + 4$  and  $y = 3x - 5$  and give the equation of the line perpendicular to  $y = 3x + 4$  which crosses the point (2,1). (2)
2. Given a 45-45-90 triangle with the height of 5cm, find the hypotenuse of the triangle. (0)
3. If  $f(x) = 2x + 8$  and  $g(x) = \frac{x}{2} - 4$ , then find the point of intersection of the two lines algebraically. (1)
4. Given a 30-60-90 triangle, if the length of its smallest side is 4 cm, then find the length of the height and the hypotenuse of the triangle. (1)

## Example Graphing Questions:

1. Solve and graph the function  $3 \geq |x + y|$  using complete solution curves. (2)
2. The equation, where  $y$  is a function of  $x$ , has the following coordinates. Is the equation a function? Explain. (1)

Y	X
1	3
2	3
3	4
4	5

3. Does there exist a cubic\* function which is even? (1)
  - a. Cubic in this case means that does there exists any constants  $a$ ,  $b$ ,  $c$ , and  $d$ , where the function  $ax^3 + bx^2 + cx + d$  will be an even function given  $a \neq 0$ .

**Example Algebraic Formulas and Method Questions:**

- Simplify\* the following expression:  $\frac{k}{2+i^3}$  (1)
  - \*Remember simplifying an expression means to annihilate any radicals in the denominator and leave any radicals in the numerator in simplest radical form.
- Simplify\* the following expression:  $\frac{k}{2+i^3-i^7}$  (3)
  - \*The question simply wants to know if you are able to remove all radicals in the denominator. Therefore for the sake of time conservation you may leave the numerator not simplified.
- Simplify the following expression:  $\frac{x^3+5x^2+6x}{x^2+2x}$ . (1)
- Find the roots of the function:  $y = x^2 + 10x + 5$  (1)
- An arrow is shot which reaches the height of 10cm and lands 5cm away from the shooter. If all else equals\* find the formula of the quadratic equation which best represents the path of the arrow. (2)
  - \*The term "if all else equals" simply means that all other variables are held constant. For example wind speed and other variables will not affect the question.
- Hamza scored a grade of 79 and 92 on his first two tests. What is the minimum score Hamza needs on his next exam to achieve a final grade of 90? Assume all exam grades are weighted equally. (1)
- Find the middle term of the expansion  $(2x + 3)^6$  (2)
- Given  $x^2 + 4x - 20 = 0$ , describe the nature of the roots. (1)

**Example Algebra Questions:**

- If  $f(x) = 3x + 8$  and  $g(x) = x^2 - 2x$ , then find  $f^{-1}(g(3))$ . (2)
- If  $\log_5 64 = n$  then find  $n$ . (0)
  - \*This problem may deserve a level of (1) in difficulty, but in this case the answer should be observably trivial after completing one step, thus given a rating of (0).
- Simplify\* the expression:  $\log_3 3^3 \sqrt[3]{81} - \log_5 27^{-6} + 3\log_8 9$ . (3)
  - \*Simplification of log expressions could in this case mean combining the logs.
  - However doing that would be extremely messy and deserving a rating higher than a (3). Rather, solve this expression such that you have an answer with no logs, thus a quantitative result.
- Find the range and domain of  $y = \sqrt{x+2}$ . (0)
- An automated tennis ball dispenser dispenses 45 ball per minute. If an automated baseball dispenser dispenses baseballs 20% faster than the tennis ball dispenser, then how fast does the baseball dispenser dispense every hour? (1)
- A car was purchased at \$45,000. After three years the car is now worth \$32,000. Find the rate of depression per year. Leave the answer to the nearest percent. (2)
- Simplify the following expression:  $\frac{(x+2)!}{x!}$  (1)

**Example Probability Questions:**

- A committee of 6 people is to be selected from a group of 10 women and 4 men. What is the probability that the committee consists of 3 women and 3 men? (2)
- Ali is a professional basketball player who has determined that he makes nine 3pt shots per every ten attempts. What is the probability that out of 25 shots he misses 4? (1)

**Example Logic Questions:**

1. How many ways can the letters of the word TEAM be arranged? (1)
2. If the operation  $a \cdot b$  is defined by  $a^b + b$ , such that  $a \cdot b = a^b + b$ , then find  $n$ , where  $n \cdot 2 = 38$ . (1)

**Answers begin below.**

If you have any questions regarding how to solve these problems, please contact [competitions@gatmistsified.com](mailto:competitions@gatmistsified.com).

**Geometry Formula Questions:**

1.  $16\pi$
2. Volume =  $1,500 \text{ cm}^3$ , Surface Area =  $1,000 \text{ cm}^2$ .
3.  $6 \text{ cm}$
4.  $r = 4$
5.  $y = \frac{-x}{2} + 7$
6.  $(5,3)$
7.  $x - 22 + (y - 4)^2 = 64$
8.  $5\sqrt{2}$
9.  $y = \frac{3x}{7} + \frac{34}{7}$

**Geometry Questions:**

1. Distance = 9 units\*,  $y = -(\frac{x}{3} + \frac{5}{3})$ . \*Since the units were not provided, distance = 9 will also be accepted. However, be sure to provide units in your answer if it is provided in your question.
2.  $5\sqrt{2} \text{ cm}$
3.  $(-\frac{36}{5}, -\frac{32}{5})$
4. Height =  $4\sqrt{3} \text{ cm}$ , hypotenuse = 8 cm

**Graphing Questions:**

1.  $3 - x \geq y \geq -3 - x$ . The graph of the inequality will look like the shaded region between the lines  $y = 3 - x$  and  $y = -3 - x$ .
2. The curve is not a function. Fails to pass the vertical line test.
3. No, by the definition of an even function.

**Algebraic Formulas and Method Questions:**

1.  $2 - \sqrt{3}$
2.  $\frac{-(2-\sqrt{5}+\sqrt{7})-3-2\sqrt{33}}{70}$
3.  $x + 3$
4.  $x = -5 \pm 2\sqrt{5}$
5.  $y = \frac{-6x^2}{5} + 8x$
6. 99%
7.  $4320x^3$
8. Real, irrational and unequal.

**Algebra Questions:**

1.  $\frac{-5}{x}$
2.  $n = 6$
3.  $\frac{147}{40}$
4.  $D: [-2, \infty), R: [0, \infty)$
5. 3240 baseballs/hour
6. rate of depression = 11%
7.  $x^2 + 3x + 2$

**Probability Questions:**

1. 16%14%

**Logic Questions**

1. 24 possible combinations
2.  $n = 6$